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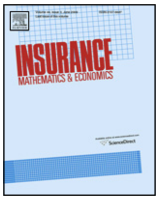

Ruin probability in the presence of interest earnings and tax payments

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ABSTRACT

In this paper we investigate the ruin probability in a general risk model driven by a compound Poisson process. We derive a formula for the ruin probability from which the Albrecher–Hipp tax identity follows as a corollary. Then we study, as an important special case, the classical risk model with a constant force of interest and loss-carried-forward tax payments. For this case we derive an exact formula for the ruin probability when the claims are exponential and an explicit asymptotic formula when the claims are subexponential.

1. Introduction

In the classical risk model, the surplus process of an insurer is described as

$$U(t) = u + ct - S(t), \quad t \geq 0.$$

Here, $u \geq 0$ is the initial surplus, $c > 0$ is the constant premium rate, and $S(t) = \sum_{i=1}^{N(t)} X_i$ is a compound Poisson process modelling aggregate claims having the Poisson parameter $\lambda > 0$ and individual claim-size distribution F_X with $F_X(0) = 0$ and mean $\mu > 0$. An important quantity in risk theory is the (infinite-time) ruin probability

$$\Psi(u) = \Pr\left(U(t) < 0 \text{ for some } t \geq 0 | U(0) = u\right).$$

Denote by $\Phi(u) = 1 - \Psi(u)$ the non-ruin probability.

Albrecher and Hipp (2007) extended the study to incorporate tax payments. They proposed a loss-carried-forward tax scheme with a constant tax rate $\gamma \in [0, 1)$. That is, tax is paid at a fixed rate $\gamma \in [0, 1)$ whenever the insurer is in a "profitable situation". The reader is referred to their paper for more details about the loss-carried-forward tax scheme. The modified surplus at time t is written as $U_\gamma(t)$ and the corresponding ruin and non-ruin probabilities are denoted by $\Psi_\gamma(u)$ and $\Phi_\gamma(u)$, respectively. Using conditioning techniques and product identities and assuming that the insurer is in a "profitable condition" immediately after time 0, they established the following remarkably simple formula:

$$\Phi_\gamma(u) = [\Phi(u)]^{\frac{1}{1-\gamma}}. \tag{1.1}$$

Subsequently, Albrecher et al. (2009) refined the proof of (1.1) by linking queueing concepts with risk theory and extended the identity to arbitrary surplus-dependent tax rates.

In this paper we are interested in the ruin probability of a general risk model whose surplus process at time t is denoted by $U_g(t)$ and characterized by the following stochastic differential

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$$\Phi_{\delta,\gamma}\,(u) = \exp\left\{-\int_u^\infty \frac{\delta\left(\frac{x}{\mu}+\frac{c}{\delta\mu}\right)^{\lambda/\delta}\mathrm{e}^{-\left(\frac{x}{\mu}+\frac{c}{\delta\mu}\right)}}{\frac{\delta}{\lambda}\left(\frac{c}{\delta\mu}\right)^{\lambda/\delta}\mathrm{e}^{-\frac{c}{\delta\mu}}+\int_0^{x/\mu}\left(y+\frac{c}{\delta\mu}\right)^{\lambda/\delta-1}\mathrm{e}^{-\left(y+\frac{c}{\delta\mu}\right)}\mathrm{d}y}\,\frac{\mathrm{d}x}{(c+\delta x)\,(1-\gamma\,(x))}\right\}.$$

Box I.

At the same time, it follows from (2.5) that, as $u \to \infty$,

$$\begin{aligned}\Psi_{\delta,\gamma}\,(u) &= 1-\exp\left\{-\int_u^\infty \frac{\lambda q(x)}{(c+\delta x)\,(1-\gamma\,(x))}\mathrm{d}x\right\}\\ &\sim \int_u^\infty \frac{\lambda q(x)}{(c+\delta x)\,(1-\gamma\,(x))}\mathrm{d}x, \qquad (2.9)\end{aligned}$$

where the symbol '$\sim$' means 'asymptotic equivalence'; see Section 4.2 below for details. Relations (2.8) and (2.9) enable us to study the ruin probability $\Psi_{\delta,\gamma}(u)$ by virtue of the existing results on the ruin probability $\Psi_\delta\,(u)$. Detailed discussions will be given in the next two sections.

3. Exponentially distributed claims

Suppose that the claim-sizes follow an exponential distribution

$$F_X\,(x) = 1-\mathrm{e}^{-x/\mu}, \quad x \ge 0,$$

for some $\mu > 0$. We obtain the first main result below:

Theorem 3.1. *Under the above assumptions, we have the formula in Box I.*

Proof. By a formula of Sundt and Teugels (1995, line 9 of page 21) with $(c/\delta)^{\lambda/\delta}$ in the last term of the denominator corrected to $(c/(\delta\mu))^{\lambda/\delta}$, we have

$$\Phi_\delta\,(u) = \frac{\Gamma\left(\frac{\lambda}{\delta},\frac{c}{\delta\mu}\right)+\frac{\delta}{\lambda}\left(\frac{c}{\delta\mu}\right)^{\lambda/\delta}\mathrm{e}^{-\frac{c}{\delta\mu}}-\Gamma\left(\frac{\lambda}{\delta},\frac{c+\delta u}{\delta\mu}\right)}{\Gamma\left(\frac{\lambda}{\delta},\frac{c}{\delta\mu}\right)+\frac{\delta}{\lambda}\left(\frac{c}{\delta\mu}\right)^{\lambda/\delta}\mathrm{e}^{-\frac{c}{\delta\mu}}},$$

where $\Gamma\,(a,x) = \int_x^\infty y^{a-1}\mathrm{e}^{-y}\mathrm{d}y$ is the (upper) incomplete gamma function. Then by (2.8), we obtain that

$$\begin{aligned}q(u) &= \frac{\Phi_\delta'\,(u)}{\Phi_\delta\,(u)}\frac{c+\delta u}{\lambda}\\ &= \frac{\frac{\delta}{\lambda}\left(\frac{c+\delta u}{\delta\mu}\right)^{\lambda/\delta}\mathrm{e}^{-\frac{c+\delta u}{\delta\mu}}}{\Gamma\left(\frac{\lambda}{\delta},\frac{c}{\delta\mu}\right)+\frac{\delta}{\lambda}\left(\frac{c}{\delta\mu}\right)^{\lambda/\delta}\mathrm{e}^{-\frac{c}{\delta\mu}}-\Gamma\left(\frac{\lambda}{\delta},\frac{c+\delta u}{\delta\mu}\right)}\\ &= \frac{\frac{\delta}{\lambda}\left(\frac{u}{\mu}+\frac{c}{\delta\mu}\right)^{\lambda/\delta}\mathrm{e}^{-\left(\frac{u}{\mu}+\frac{c}{\delta\mu}\right)}}{\frac{\delta}{\lambda}\left(\frac{c}{\delta\mu}\right)^{\lambda/\delta}\mathrm{e}^{-\frac{c}{\delta\mu}}+\int_0^{u/\mu}\left(y+\frac{c}{\delta\mu}\right)^{\lambda/\delta-1}\mathrm{e}^{-\left(y+\frac{c}{\delta\mu}\right)}\mathrm{d}y}. \qquad (3.1)\end{aligned}$$

Thus, the formula in Box I is obtained by substituting (3.1) into (2.5). □

4. Subexponential claims

4.1. Subexponential distributions

One of the most important classes of heavy-tailed distributions is the subexponential class $\mathcal{S}$. By definition, a distribution F on $[0,\infty)$ is said to be subexponential, if $\overline{F}(x) = 1-F(x) > 0$ for all $x \ge 0$ and the relation

$$\lim_{x\to\infty}\frac{\overline{F^{n*}}(x)}{\overline{F}(x)} = n$$

holds for some (or, equivalently, for all) $n = 2, 3, \ldots$, where F^{n*} denotes the n-fold convolution of F. Closely related is the class $\mathcal{L}$ of long-tailed distributions, characterized by the relation

$$\lim_{x\to\infty}\frac{\overline{F}(x+y)}{\overline{F}(x)} = 1$$

for some (or, equivalently, for all) $y > 0$. It is well known that $\mathcal{S} \subset \mathcal{L}$. The class $\mathcal{S}$ is often used to model claim-size distributions; see, for instance, Embrechts and Veraverbeke (1982), Embrechts et al. (1997), Schmidli (2005), Tang (2005) and Hao and Tang (2008), among others.

Let us recall a distributional index. For a distribution F on $[0,\infty)$ with $\overline{F}\,(x) > 0$ for all $x \ge 0$, define

$$J_*\,(F) = \sup\left\{-\frac{\ln \overline{F}^*\,(v)}{\ln v} : v > 1\right\} \quad \text{with}$$

$$\overline{F}^*\,(v) = \limsup_{x\to\infty}\frac{\overline{F}\,(vx)}{\overline{F}\,(x)} \quad \text{for } v > 1. \qquad (4.1)$$

Following Tang and Tsitsiashvili (2003), we call the quantity $J_*\,(F)$ the lower Matuszewska index of the function $1/\overline{F}$. Suppose $0 < J_*\,(F) \le \infty$, by Tang and Tsitsiashvili (2003), who attributed this to Bingham et al. (1987), for each $0 < m < J_*\,(F)$, there are positive constants C and x_0 such that the inequality

$$\frac{\overline{F}\,(xy)}{\overline{F}\,(x)} \le Cy^{-m} \qquad (4.2)$$

holds uniformly for $xy \ge x \ge x_0$.

A useful subclass of $\mathcal{S}$ is the class $\mathcal{A}$. A distribution F on $[0,\infty)$ is said to belong to the class $\mathcal{A}$ if $F \in \mathcal{S}$ and $0 < J_*\,(F) \le \infty$. The class $\mathcal{A}$ was first introduced by Konstantinides et al. (2002), who pointed out that this class covers almost all popular subexponential distributions. Recent studies on the class $\mathcal{A}$ can be found in Tang (2006) and the references therein.

4.2. Second main result

Hereafter, all limit relationships are for $u \to \infty$ unless otherwise stated. Let f_1 and f_2 be two positive functions satisfying

$$C_* \le \liminf\frac{f_1(u)}{f_2(u)} \le \limsup\frac{f_1(u)}{f_2(u)} \le C^*.$$

We write $f_1 = O\,(f_2)$ if $C^* < \infty$, $f_1 = o\,(f_2)$ if $C^* = 0$, and $f_1 \sim f_2$ if $C^* = C_* = 1$.

For a distribution F on $[0,\infty)$ with finite mean $\mu > 0$, its equilibrium distribution is defined as

$$F_e\,(x) = \frac{1}{\mu}\int_0^x \overline{F}\,(y)\,\mathrm{d}y, \quad x \ge 0.$$

The second main result is given below:

Theorem 4.1. *Suppose that both the claim-size distribution F_X and its equilibrium distribution $F_{X,e}$ are subexponential, and that $J_*\,(F_X)$ defined by (4.1) satisfies $1 < J_*\,(F_X) \le \infty$. Then*

$$\Psi_{\delta,\gamma}\,(u) \sim \int_u^\infty \frac{\lambda\overline{F_X}\,(x)}{(c+\delta x)\,(1-\gamma\,(x))}\mathrm{d}x. \qquad (4.3)$$

When $\gamma(x) \equiv 0$ and $\delta > 0$, (4.3) coincides with Theorem 2.1 of Konstantinides et al. (2002); see also Lemma 4.1(i) below. For $\gamma(x) \equiv \gamma \in [0, 1)$ and $\delta > 0$, an immediate consequence of (2.7) is

$$\Psi_{\delta,\gamma}(u) = 1 - (1 - \Psi_\delta(u))^{\frac{1}{1-\gamma}} \sim \frac{1}{1-\gamma}\Psi_\delta(u). \tag{4.4}$$

Under the conditions of Theorem 4.1, the asymptotic relation (4.4) can be extended to the case of surplus-dependent tax rates. Actually, if we additionally assume that $\lim_{x\to\infty}\gamma(x) \equiv \gamma \in [0, 1)$ then

$$\Psi_{\delta,\gamma}(u) \sim \frac{1}{1-\gamma}\int_u^\infty \frac{\lambda\overline{F_X}(x)}{c+\delta x}\mathrm{d}x \sim \frac{1}{1-\gamma}\Psi_\delta(u),$$

where the second asymptotic relation is again due to Lemma 4.1(i) given below.

4.3. *Proof of* Theorem 4.1

We need two lemmas.

Lemma 4.1. *For $\gamma(x) \equiv 0$, if $J_*(F_X)$ defined by (4.1) satisfies $1 < J_*(F_X) \le \infty$, and $F_{X,e} \in \mathscr{S}$, then we have*

(i) $\Psi_\delta(u) \sim \frac{\lambda}{\delta}\int_u^\infty \frac{\overline{F_X}(x)}{x}\mathrm{d}x$;

(ii) $\Psi_\delta(u) = O\left(\overline{F_X}(u)\right)$.

Proof. (i) It follows from inequality (4.2) that for each $1 < m < J_*(F_X)$, there is a positive constant C such that

$$\frac{\overline{F_{X,e}}(vx)}{\overline{F_{X,e}}(x)} = \frac{v\int_x^\infty \overline{F_X}(vy)\,\mathrm{d}y}{\int_x^\infty \overline{F_X}(y)\,\mathrm{d}y} = \frac{v\int_x^\infty \frac{\overline{F_X}(vy)}{\overline{F_X}(y)}\overline{F_X}(y)\,\mathrm{d}y}{\int_x^\infty \overline{F_X}(y)\,\mathrm{d}y} \le vCv^{-m} < 1$$

holds for large v. This implies $F_{X,e} \in \mathscr{A}$. Thus, statement (i) is obtained immediately by Theorem 2.1 of Konstantinides et al. (2002).

(ii) By statement (i) and inequality (4.2), it is clear that for each $1 < m < J_*(F_X)$, there is a positive constant $\tilde{C}$ such that

$$\frac{\Psi_\delta(u)}{\overline{F_X}(u)} \sim \frac{\lambda}{\delta}\int_u^\infty \frac{\overline{F_X}(x)}{\overline{F_X}(u)}\frac{1}{x}\mathrm{d}x \le \frac{\lambda\tilde{C}}{\delta}\int_u^\infty \left(\frac{x}{u}\right)^{-m}\frac{1}{x}\mathrm{d}x = \frac{\lambda\tilde{C}}{\delta m} < \infty.$$

This implies that the relation $\Psi_\delta(u) = O\left(\overline{F_X}(u)\right)$ holds. □

The next lemma is well known. For proofs see Embrechts and Goldie (1980), Cline (1986, Corollary 1), or Tang and Tsitsiashvili (2003, Lemma 3.2).

Lemma 4.2. *Let $F = F_1 * F_2$ be the convolution of two distributions F_1 and F_2. If $F_1 \in \mathscr{L}$, $F_2 \in \mathscr{S}$, and $\overline{F_1}(u) = O\left(\overline{F_2}(u)\right)$, then $F \in \mathscr{S}$ and $\overline{F}(u) \sim \overline{F_1}(u) + \overline{F_2}(u)$.*

Now, we are ready to give:

Proof of Theorem 4.1. As shown in Sundt and Teugels (1995, (1)), we have

$$\frac{c+\delta u}{\lambda}\Phi'_\delta(u) = \Phi_\delta(u) - \int_0^u \Phi_\delta(u-x)\,\mathrm{d}F_X(x). \tag{4.5}$$

It follows from (2.8) and (4.5) that

$$\begin{aligned} q(u) &= \frac{\Phi'_\delta(u)}{\Phi_\delta(u)}\frac{c+\delta u}{\lambda} \\ &= 1 - \int_0^u \frac{\Phi_\delta(u-x)}{\Phi_\delta(u)}\mathrm{d}F_X(x) \\ &= \overline{F_X}(u) + \int_0^u \left(1 - \frac{\Phi_\delta(u-x)}{\Phi_\delta(u)}\right)\mathrm{d}F_X(x). \end{aligned} \tag{4.6}$$

Clearly,

$$\begin{aligned} &\int_0^u \left(1 - \frac{\Phi_\delta(u-x)}{\Phi_\delta(u)}\right)\mathrm{d}F_X(x) \\ &\sim \int_0^u (\Phi_\delta(u) - \Phi_\delta(u-x))\,\mathrm{d}F_X(x) \\ &= \int_0^u \Psi_\delta(u-x)\,\mathrm{d}F_X(x) - \Psi_\delta(u)F_X(u) \\ &= \overline{\Phi_\delta * F_X}(u) - \overline{F_X}(u) - \Psi_\delta(u)F_X(u) \\ &\le \overline{\Phi_\delta * F_X}(u) - \left(\Psi_\delta(u) + \overline{F_X}(u)\right)F_X(u). \end{aligned} \tag{4.7}$$

Under the given conditions, by Lemma 4.1(i), it is easy to verify that $\Phi_\delta \in \mathscr{L}$. Thus, by Lemma 4.1(ii) and Lemma 4.2, we have $\Phi_\delta * F_X \in \mathscr{S}$ and

$$\overline{\Phi_\delta * F_X}(u) \sim \Psi_\delta(u) + \overline{F_X}(u). \tag{4.8}$$

Substituting (4.8) into (4.7) and using Lemma 4.1(ii) again, we obtain that

$$\begin{aligned} \int_0^u \left(1 - \frac{\Phi_\delta(u-x)}{\Phi_\delta(u)}\right)\mathrm{d}F_X(x) &= o\left(\Psi_\delta(u) + \overline{F_X}(u)\right) \\ &= o\left(\overline{F_X}(u)\right). \end{aligned} \tag{4.9}$$

Combining (4.6) with (4.9) yields that

$$q(u) \sim \overline{F_X}(u). \tag{4.10}$$

Thus, relation (4.3) follows from (2.9) and (4.10). □

5. Numerical results

In this section we test the accuracy of the asymptotic formula (4.3). For this purpose, we choose Pareto, log-normal, and Weibull distributions, respectively, as the claim-size distribution F_X. Suppose that $c = 1.1$, $\lambda = 1$, $\delta = 0.05$, and

$$\gamma(x) = \begin{cases} 0.10, & 0 < x \le 10^4, \\ 0.18, & 10^4 < x \le 10^5, \\ 0.30, & 10^5 < x \le 10^6, \\ 0.50, & x > 10^6. \end{cases}$$

We have made these selections just for our verification purpose.

(i) The Pareto distribution is of the form

$$F_X(x) = 1 - \left(\frac{\theta}{x+\theta}\right)^\alpha, \quad x \ge 0, \theta > 0, \text{and } \alpha > 1.$$

Choose $\theta = 1$ and $\alpha = 2$, so that X has mean $\mu = 1$. It follows from (4.3) that

$$\begin{aligned} \Psi_{\delta,\gamma}(u) \sim \Bigg(&\int_0^{10^4-u}\frac{1}{0.9} + \int_{10^4-u}^{10^5-u}\frac{1}{0.82} + \int_{10^5-u}^{10^6-u}\frac{1}{0.7} \\ &+ \int_{10^6-u}^\infty \frac{1}{0.5}\Bigg)\frac{(x+u+1)^{-2}}{1.1+0.05(x+u)}\mathrm{d}x. \end{aligned}$$

(ii) The log-normal distribution has the probability density function

$$f_X(x; \theta, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}}\mathrm{e}^{-\frac{(\ln x-\theta)^2}{2\sigma^2}},$$
$$x > 0, -\infty < \theta < \infty, \text{and } \sigma > 0.$$

Table 1
Numerical results for Pareto case.

u	$a =$ asymptotics	$s =$ simulation	a/s	$\lvert s-a\rvert/s$
5	0.0423911	0.0403181	1.051416113	0.051416113
10	0.0238937	0.0228016	1.047895762	0.047895762
15	0.0154625	0.0150159	1.029741807	0.029741807
20	0.0108663	0.0107063	1.014944472	0.014944472
25	0.0080707	0.00800102	1.00870889	0.00870889
30	0.00623841	0.00619607	1.006833364	0.006833364
35	0.00497017	0.00494193	1.005714367	0.005714367
40	0.00405496	0.00404501	1.002459821	0.002459821
45	0.00337236	0.00336562	1.002002603	0.002002603
50	0.00284941	0.00284397	1.001912819	0.001912819

Table 2
Numerical results for log-normal case.

u	$a =$ asymptotics	$s =$ simulation	a/s	$\lvert s-a\rvert/s$
5	0.0430707	0.0399102	1.079190282	0.079190282
10	0.0216612	0.0202162	1.07147733	0.07147733
15	0.0125155	0.0118897	1.052633792	0.052633792
20	0.00789262	0.00750162	1.052122075	0.052122075
25	0.00529126	0.00509896	1.037713573	0.037713573
30	0.00371263	0.00361432	1.027200137	0.027200137
35	0.00269896	0.00262899	1.026614784	0.026614784
40	0.0020187	0.00199768	1.010522206	0.010522206
45	0.00154569	0.00153231	1.008731915	0.008731915
50	0.001207	0.00120125	1.004786681	0.004786681

Table 3
Numerical results for Weibull case.

u	$a =$ asymptotics	$s =$ simulation	a/s	$\lvert s-a\rvert/s$
5	0.0370033	0.0382785	0.96668626	0.03331374
10	0.0138678	0.01401246	0.989676331	0.010323669
15	0.00593535	0.00597658	0.993101406	0.006898594
20	0.00277967	0.00278196	0.999176839	0.000823161
25	0.00139133	0.00139152	0.999863459	0.000136541
30	0.000733461	0.000733503	0.999942741	5.72595E−05
35	0.000403191	0.000403211	0.999950398	4.96018E−05
40	0.000229479	0.000229483	0.99998257	1.74305E−05
45	0.000134514	0.000134516	0.999985132	1.48681E−05
50	8.08758E−05	8.08764E−05	0.999992581	7.41873E−06

Choose $\theta = -1$ and $\sigma = \sqrt{2}$, so that X has mean $\mu = 1$. It also follows from (4.3) that

$$\Psi_{\delta,\gamma}(u) \sim \left(\int_0^{10^4-u}\frac{1}{0.9}+\int_{10^4-u}^{10^5-u}\frac{1}{0.82}+\int_{10^5-u}^{10^6-u}\frac{1}{0.7}+\int_{10^6-u}^{\infty}\frac{1}{0.5}\right)\frac{\int_{x+u}^{\infty}\frac{1}{2\sqrt{\pi y}}\mathrm{e}^{-\frac{(\ln y+1)^2}{4}}\mathrm{d}y}{1.1+0.05(x+u)}\mathrm{d}x.$$

(iii) The Weibull distribution is of the form

$$F_X(x) = 1-\mathrm{e}^{-(x/\theta)^{\alpha}}, \quad x \geq 0, \theta > 0, \text{ and } 0 < \alpha < 1.$$

Choose $\theta = \alpha = 0.5$, so that X has mean $\mu = 1$. It again follows from (4.3) that

$$\Psi_{\delta,\gamma}(u) \sim \left(\int_0^{10^4-u}\frac{1}{0.9}+\int_{10^4-u}^{10^5-u}\frac{1}{0.82}+\int_{10^5-u}^{10^6-u}\frac{1}{0.7}+\int_{10^6-u}^{\infty}\frac{1}{0.5}\right)\frac{\mathrm{e}^{-\sqrt{2(x+u)}}}{1.1+0.05(x+u)}\mathrm{d}x.$$

For the three cases, numerical results are obtained by the packages Mathematica 6.0 and Matlab 6.5. They are copied to Tables 1–3, respectively, where E-05 means 10^{-5} and E-06 means 10^{-6} in Table 3.

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References

Albrecher, H., Borst, S., Boxma, O., Resing, J., 2009. The tax identity in risk theory — a simple proof and an extension. Insurance: Mathematics and Economics 44 (2), 304–306.

Albrecher, H., Hipp, C., 2007. Lundberg's risk process with tax. Blätter der DGVFM 28 (1), 13–28.

Bingham, N.H., Goldie, C.M., Teugels, J.L., 1987. Regular Variation. Cambridge University Press, Cambridge.

Cline, D.B.H., 1986. Convolution tails, product tails and domains of attraction. Probability Theory and Related Fields 72 (4), 529–557.

Embrechts, P., Goldie, C.M., 1980. On closure and factorization properties of subexponential and related distributions. Journal of the Australian Mathematical Society, Series A 29 (2), 243–256.

Embrechts, P., Klüppelberg, C., Mikosch, T., 1997. Modelling Extremal Events for Insurance and Finance. Springer-Verlag, Berlin.

Embrechts, P., Veraverbeke, N., 1982. Estimates for the probability of ruin with special emphasis on the possibility of large claims. Insurance: Mathematics and Economics 1 (1), 55–72.

Hao, X., Tang, Q., 2008. A uniform asymptotic estimate for discounted aggregate claims with subexponential tails. Insurance: Mathematics and Economics 43 (1), 116–120.

Konstantinides, D., Tang, Q., Tsitsiashvili, G., 2002. Estimates for the ruin probability in the classical risk model with constant interest force in the presence of heavy tails. Insurance: Mathematics and Economics 31 (3), 447–460.

Schmidli, H., 2005. On optimal investment and subexponential claims. Insurance: Mathematics and Economics 36 (1), 25–35.

Sundt, B., Teugels, J.L., 1995. Ruin estimates under interest force. Insurance: Mathematics and Economics 16 (1), 7–22.

Tang, Q., 2005. The finite-time ruin probability of the compound Poisson model with constant interest force. Journal of Applied Probability 42 (3), 608–619.

Tang, Q., 2006. The subexponentiality of products revisited. Extremes 9 (3–4), 231–241.

Tang, Q., Tsitsiashvili, G., 2003. Precise estimates for the ruin probability in finite horizon in a discrete-time model with heavy-tailed insurance and financial risks. Stochastic Processes and their Applications 108 (2), 299–325.