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Ruin probability of the renewal model with risky investment and large claims

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Abstract The ruin probability of the renewal risk model with investment strategy for a capital market index is investigated in this paper. For claim sizes with common distribution of extended regular variation, we study the asymptotic behaviour of the ruin probability. As a corollary, we establish a simple asymptotic formula for the ruin probability for the case of Pareto-like claims.

Keywords: asymptotics, extended regular variation, renewal risk model, risky investment strategy, ruin probability

MSC(2000): 60G70, 60K30, 60K37

1 Introduction and model

Consider a continuous-time renewal risk model, in which claim sizes X_k, $k = 1, 2, \ldots$, constitute a sequence of independent, identically distributed (i.i.d.), and nonnegative random variables with generic random variable X distributed by F, while their arrival times $0 = \tau_0 < \tau_1 < \tau_2 < \cdots$ constitute an ordinary renewal counting process

$$N_t = \sum_{k=1}^{\infty} 1_{(0,t]}(\tau_k), \qquad t \geqslant 0, \tag{1}$$

where $1_{(0,t]}(x)$ denotes an indicator function whose value is 1 if $x \in (0, t]$ and 0 otherwise. Assume that $\{X_k, k = 1, 2, \ldots\}$ and $\{N_t, t \geqslant 0\}$ are mutually independent. This renewal risk model is extended by allowing investments. Suppose that a portion of the surplus is invested into a Black-Scholes type market index whose price process is modeled by a geometric Brownian motion. Thus the wealth process $\{U_t, t \geqslant 0\}$ of the insurer can be defined by the stochastic differential equation

$$\mathrm{d}U_t = c\mathrm{d}t - \mathrm{d}S_t + \theta U_t(\mu \mathrm{d}t + \sigma \mathrm{d}W_t), \tag{2}$$

where $c > 0$ is the constant premium rate, $S_t = \sum_{k=1}^{N_t} X_k$ represents aggregate claims during the time interval $(0, t]$, $\mu > 0$ and $\sigma > 0$ are two known parameters, $W = \{W_t, t \geqslant 0\}$ is a standard Brownian motion independent of $\{X_k, k = 1, 2, \ldots\}$ and $\{N_t, t \geqslant 0\}$, and $\theta \in [0, 1]$ is the constant fraction of the surplus invested into the risky asset. Such a strategy is dynamic in the sense that it requires an instantaneous rebalancing of the portfolio according to price changes.

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The solution of (2) is given by

$$U_t = \exp\{\tilde{\mu}t + \tilde{\sigma}W_t\}[u + \int_0^t \exp\{-\tilde{\mu}v - \tilde{\sigma}W_v\}(c\mathrm{d}v - \mathrm{d}S_v)], \tag{3}$$

as is easily derived by Itô's formula, where $U_0 = u > 0$ is the initial surplus, $\tilde{\mu} = \mu\theta - \sigma^2\theta^2/2$, and $\tilde{\sigma} = \sigma\theta$. We intend to study the asymptotic behaviour of the ruin probability of the renewal risk model (3) in the sequel.

Calculation of finite- and infinite-time ruin probabilities in the continuous- and discrete-time renewal risk models has been done by numerous authors; see, for instance, [1–5], among others. Enormous attention has also been paid to the case of large claims in the presence of investments in the literature; see, for instance, [6–10].

The rest of this paper is organized as follows. Section 2 defines ruin probability and other concepts, Section 3 presents the main result, and Section 4 provides the proof.

2 Definitions

Definition 2.1. *The ruin probability of the renewal risk model* (3) *is defined to be*

$$\Psi(u) = \Pr(U_t < 0, \exists\, t \geqslant 0 \mid U_0 = u), \qquad u \geqslant 0. \tag{4}$$

Definition 2.2. *A distribution F on $[0,\infty)$ is said to belong to the class $\mathcal{R}_{-\alpha}$ for some constant $\alpha > 0$ if $\overline{F}(x) = 1 - F(x) > 0$ for all $x \geqslant 0$ and*

$$\lim_{x\to\infty} \frac{\overline{F}(vx)}{\overline{F}(x)} = v^{-\alpha} \qquad \text{for all } v \geqslant 1. \tag{5}$$

The class $\mathcal{R}$ is the union of all classes $\mathcal{R}_{-\alpha}$ over the range of $\alpha > 0$.

Definition 2.3. *A distribution F on $[0,\infty)$ is said to belong to the class* $\mathrm{ERV}(-\alpha,-\beta)$ *for some constants α and β, $0 < \alpha \leqslant \beta < \infty$, if $\overline{F}(x) > 0$ for all $x \geqslant 0$ and*

$$v^{-\beta} \leqslant \liminf_{x\to\infty} \frac{\overline{F}(vx)}{\overline{F}(x)} \leqslant \limsup_{x\to\infty} \frac{\overline{F}(vx)}{\overline{F}(x)} \leqslant v^{-\alpha} \qquad \text{for all } v \geqslant 1. \tag{6}$$

The class ERV *is the union of all classes* $\mathrm{ERV}(-\alpha,-\beta)$ *over the range of* $0 < \alpha \leqslant \beta < \infty$.

The extended regular variation described by (6) has been used in the study of precise large deviations by many people; see [11–13], etc. It is well known that ERV is a subclass of the class $\mathcal{S}$ (see Theorem 1 of [14]), where $\mathcal{S}$ is the class of subexponential distributions characterized by the relations $\overline{F}(x) > 0$ for all $x \geqslant 0$ and $\lim_{x\to\infty} \overline{F^{2*}}(x)/\overline{F}(x) = 2$ ($F^{2*}(x)$ denotes the 2-fold convolution of $F(x)$). It is usually easier to deal with distributions from the class $\mathcal{R}$ than those from the class ERV because of the well-developed Karamata theory. Although the class ERV is marginally larger than the class $\mathcal{R}$, we expect that asymptotic results for the class ERV can provide more insight to the study in the subexponential case. For more details of heavy-tailed distributions, the reader is referred to [15, 16].

3 Main result

From now on, all limit relationships are for $u \to \infty$ unless otherwise stated. For two positive functions $f(\cdot)$ and $g(\cdot)$, we write $f(u) \sim g(u)$ if $\lim f(u)/g(u) = 1$, write $f(u) \gtrsim g(u)$ if $\liminf f(u)/g(u) \geqslant 1$, write $f(u) \lesssim g(u)$ if $\limsup f(u)/g(u) \leqslant 1$, and write $f(u) = O(g(u))$ if $\limsup f(u)/g(u) < \infty$.

Consider the renewal risk model (3). We assume throughout this paper that $F \in \mathrm{ERV}(-\alpha, -\beta)$, $1 < \alpha \leqslant \beta < \infty$, and that

$$\frac{2\tilde{\mu}}{\tilde{\sigma}^2} > \beta. \tag{7}$$

Assumption (7) reflects that the volatility should be dominated by the drift; otherwise, large volatility will lead to the bankruptcy with probability one. See [7, 17] for related discussion.

For later use, let $\xi_k = \tau_k - \tau_{k-1}$, $k = 1, 2, \ldots$, denote the i.i.d. inter-occurrence times with common distribution G on $(0, \infty)$. Recalling (1), the expectation of N_t is

$$\lambda_t = \mathrm{E}N_t = \sum_{k=1}^{\infty} \Pr(\tau_k \leqslant t) = \sum_{k=1}^{\infty} G^{k*}(t), \qquad t \geqslant 0, \tag{8}$$

where $G^{k*}(t)$ denotes the k-fold convolution of $G(t)$. λ_t defined by (8) is called the renewal function.

Thus, the main result of this paper is given as follows:

Theorem 3.1. *If $F \in \mathrm{ERV}(-\alpha, -\beta)$ for some $1 < \alpha \leqslant \beta < \infty$ and Assumption* (7) *holds then the relation*

$$\Psi(u) \sim \int_0^{\infty} \int_0^{\infty} \overline{F}(uy) \mathrm{d}\mathrm{LN}(y; \tilde{\mu}t, \tilde{\sigma}^2 t) \mathrm{d}\lambda_t \tag{9}$$

stands, where $\mathrm{LN}(y; a, b^2)$ denotes the lognormal distribution with parameters a and b^2.

Corollary 3.1. *Assume the conditions of Theorem* 3.1. *When $\beta = \alpha$, i.e. $F \in \mathcal{R}_{-\alpha}$ with some $\alpha > 0$, we have*

$$\Psi(u) \sim \overline{F}(u) \frac{\widehat{G}(\tilde{\mu}\alpha - \frac{1}{2}\tilde{\sigma}^2\alpha^2)}{1 - \widehat{G}(\tilde{\mu}\alpha - \frac{1}{2}\tilde{\sigma}^2\alpha^2)}, \tag{10}$$

where $\widehat{G}$ denotes the Laplace transform of the distribution G.

The proofs of Theorem 3.1 and Corollary 3.1 are left to Section 4.

4 Proofs

4.1 Lemmas

Consider the renewal risk model (3). Introduce

$$Y_k = \exp\{-\tilde{\mu}(\tau_k - \tau_{k-1}) - \tilde{\sigma}(W_{\tau_k} - W_{\tau_{k-1}})\}, \quad k = 1, 2, \ldots, \tag{11}$$

which are i.i.d. random variables with the same distribution of $Y_1 = \exp\{-\tilde{\mu}\xi_1 - \tilde{\sigma}W_{\xi_1}\}$.

Lemma 4.1. *Consider the claims $\{X_k, k = 1, 2, \ldots\}$ and the sequence $\{Y_k, k = 1, 2, \ldots\}$ introduced in* (11). *If $F \in \mathrm{ERV}(-\alpha, -\beta)$ for some $1 < \alpha \leqslant \beta < \infty$ then*

$$\Pr\left(\sum_{k=1}^{\infty} X_k \prod_{i=1}^{k} Y_i > u\right) \sim \sum_{k=1}^{\infty} \Pr\left(X_k \prod_{i=1}^{k} Y_i > u\right). \tag{12}$$

Proof. Note that $\{X_k, k = 1, 2, \ldots\}$ and $\{Y_k, k = 1, 2, \ldots\}$ are independent and that, under the assumption (7), $\mathrm{E}[Y_1^{\alpha-\delta} \vee Y_1^{\beta+\delta}] < 1$ for some $0 < \delta < \alpha$, we have (12) as a straightforward corollary of Theorem 3.1 of [9].

Lemma 4.2. *Let X and Y be two independent and nonnegative random variables, where X has a distribution $F \in \mathrm{ERV}(-\alpha, -\beta)$ for some $1 < \alpha \leqslant \beta < \infty$ and $\mathrm{E}Y^{\beta+\delta} < \infty$ for some $0 < \delta < \alpha$. Then for some constant $D > 0$,*

$$\liminf \frac{\Pr(XY > u)}{\Pr(X > u)} \geqslant D\mathrm{E}[Y^{\alpha-\delta} \wedge Y^{\beta+\delta}]. \tag{13}$$

Proof. See Theorem 3.5 of [18].

Lemma 4.3. *If $F \in \mathrm{ERV}(-\alpha, -\beta)$ for some $1 < \alpha \leqslant \beta < \infty$ then it holds for every $0 < \delta < \infty$, some constant $\widetilde{D} > 0$ and all large $u > 0$ that*

$$\overline{F}(u) \geqslant \widetilde{D} u^{-\beta-\delta}. \tag{14}$$

Proof. See Lemma 3.1 of [13].

4.2 Proof of Theorem 3.1

Starting with (4) and using (3), we have

$$\Psi(u) = \Pr\left(u + c\int_0^t \exp\{-\tilde{\mu}v - \tilde{\sigma}W_v\}\mathrm{d}v - \sum_{k=1}^{N_t} X_k \exp\{-\tilde{\mu}\tau_k - \tilde{\sigma}W_{\tau_k}\} < 0,\, \exists\, t \geqslant 0\right).$$

Writing $\Delta = c\int_0^\infty \exp\{-\tilde{\mu}v - \tilde{\sigma}W_v\}\mathrm{d}v$ and recalling (11), it is easy to see that

$$\Pr\left(\sum_{k=1}^\infty X_k \prod_{i=1}^k Y_i > u + \Delta\right) \leqslant \Psi(u) \leqslant \Pr\left(\sum_{k=1}^\infty X_k \prod_{i=1}^k Y_i > u\right). \tag{15}$$

Firstly, consider the upper bound. With Assumption (7), from (15), (12) and (11), we have

$$\begin{aligned}\Psi(u) &\lesssim \sum_{k=1}^\infty \Pr\left(X_k \prod_{i=1}^k Y_i > u\right)\\ &= \sum_{k=1}^\infty \Pr(X_k \exp\{-\tilde{\mu}\tau_k - \tilde{\sigma}W_{\tau_k}\} > u)\\ &= \sum_{k=1}^\infty \int_0^\infty \Pr(X\exp\{-\tilde{\mu}t - \tilde{\sigma}W_t\} > u)\mathrm{d}G^{k*}(t)\\ &= \int_0^\infty \int_0^\infty \overline{F}(uy)\mathrm{dLN}(y; \tilde{\mu}t, \tilde{\sigma}^2 t)\mathrm{d}\lambda_t,\end{aligned} \tag{16}$$

where $\mathrm{LN}(y; \tilde{\mu}t, \tilde{\sigma}^2 t)$ denotes the lognormal distribution with parameters $\tilde{\mu}t$ and $\tilde{\sigma}^2 t$.

Secondly, consider the lower bound. By (15), for arbitrarily small $\varepsilon > 0$, it holds that

$$\Psi(u) \geqslant \Pr\left(\sum_{k=1}^\infty X_k \prod_{i=1}^k Y_i > (1+\varepsilon)u\right) - \Pr(\Delta > \varepsilon u). \tag{17}$$

Employing Theorem 2.1(a) of [19], we obtain that the second term on the right-hand side of (17), for some $\beta < \beta' < 2\tilde{\mu}/\tilde{\sigma}^2$, satisfies that

$$\Pr(\Delta > \varepsilon u) \leqslant \frac{\mathrm{E}\Delta^{\beta'}}{\varepsilon^{\beta'} u^{\beta'}} = O(u^{-\beta'}). \tag{18}$$

For the first term on the right-hand side of (17), following (16) we have

$$\Pr\left(\sum_{k=1}^{\infty} X_k \prod_{i=1}^{k} Y_i > (1+\varepsilon)u\right) \sim \int_0^{\infty} \Pr(X\exp\{-\tilde{\mu}t - \tilde{\sigma}W_t\} > (1+\varepsilon)u)\mathrm{d}\lambda_t. \tag{19}$$

Let $\rho > 0$ be such that $\beta'(1-\rho) > \beta$. By the definition in (6), we continue to derive the lower bound for (19) as

$$\begin{aligned}
&\int_0^{\infty} \Pr(X\exp\{-\tilde{\mu}t - \tilde{\sigma}W_t\} > (1+\varepsilon)u,\ \exp\{-\tilde{\mu}t - \tilde{\sigma}W_t\} \leqslant u^{1-\rho})\mathrm{d}\lambda_t \\
&\quad= \int_0^{\infty} \mathrm{E}[\overline{F}((1+\varepsilon)u\exp\{\tilde{\mu}t + \tilde{\sigma}W_t\})1_{(\exp\{-\tilde{\mu}t - \tilde{\sigma}W_t\} \leqslant u^{1-\rho})}]\mathrm{d}\lambda_t \\
&\quad\gtrsim (1+\varepsilon)^{-\beta} \int_0^{\infty} \mathrm{E}[\overline{F}(u\exp\{\tilde{\mu}t + \tilde{\sigma}W_t\})1_{(\exp\{-\tilde{\mu}t - \tilde{\sigma}W_t\} \leqslant u^{1-\rho})}]\mathrm{d}\lambda_t \\
&\quad\geqslant (1+\varepsilon)^{-\beta}\bigg[\int_0^{\infty} \Pr(X\exp\{-\tilde{\mu}t - \tilde{\sigma}W_t\} > u)\mathrm{d}\lambda_t \\
&\qquad\qquad - \int_0^{\infty} \Pr(\exp\{-\tilde{\mu}t - \tilde{\sigma}W_t\} > u^{1-\rho})\mathrm{d}\lambda_t\bigg].
\end{aligned} \tag{20}$$

Next, we prove that the second integral of (20) is asymptotically negligible when compared with the first integral of (20). By (13), it holds for some $0 < \delta < \alpha$ and some constant $D > 0$ that

$$\begin{aligned}
&\int_0^{\infty} \Pr(X\exp\{-\tilde{\mu}t - \tilde{\sigma}W_t\} > u)\mathrm{d}\lambda_t \\
&\quad= \overline{F}(u) \int_0^{\infty} \frac{\Pr(X\exp\{-\tilde{\mu}t - \tilde{\sigma}W_t\} > u)}{\Pr(X > u)}\mathrm{d}\lambda_t \\
&\quad\geqslant D\overline{F}(u) \int_0^{\infty} \mathrm{E}[\exp\{-(\alpha-\delta)(\tilde{\mu}t + \tilde{\sigma}W_t)\} \wedge \exp\{-(\beta+\delta)(\tilde{\mu}t + \tilde{\sigma}W_t)\}]\mathrm{d}\lambda_t.
\end{aligned} \tag{21}$$

It is easy to check that the integral in (21) is a finite constant. Recalling Assumption (7), for the β' used above we have

$$\begin{aligned}
\int_0^{\infty} \Pr(\exp\{-\tilde{\mu}t - \tilde{\sigma}W_t\} > u^{1-\rho})\mathrm{d}\lambda_t &\leqslant \int_0^{\infty} \frac{\mathrm{E}[\exp\{-\beta'(\tilde{\mu}t + \tilde{\sigma}W_t)\}]}{u^{\beta'(1-\rho)}}\mathrm{d}\lambda_t \\
= u^{-\beta'(1-\rho)} \int_0^{\infty} \exp\left\{-\left(\tilde{\mu}\beta' - \frac{1}{2}\tilde{\sigma}^2\beta'^2\right)t\right\}\mathrm{d}\lambda_t &= u^{-\beta'(1-\rho)} \frac{\widehat{G}(\tilde{\mu}\beta' - \frac{1}{2}\tilde{\sigma}^2\beta'^2)}{1 - \widehat{G}(\tilde{\mu}\beta' - \frac{1}{2}\tilde{\sigma}^2\beta'^2)}.
\end{aligned} \tag{22}$$

Following from (21), (22) and (14), we obtain

$$\lim \frac{\int_0^{\infty} \Pr(\exp\{-\tilde{\mu}t - \tilde{\sigma}W_t\} > u^{1-\rho})\mathrm{d}\lambda_t}{\int_0^{\infty} \Pr(X\exp\{-\tilde{\mu}t - \tilde{\sigma}W_t\} > u)\mathrm{d}\lambda_t} = 0. \tag{23}$$

Therefore, the combination of (19), (20) and (23) gives that

$$\Pr\left(\sum_{k=1}^{\infty} X_k \prod_{i=1}^{k} Y_i > (1+\varepsilon)u\right) \gtrsim (1+\varepsilon)^{-\beta} \int_0^{\infty} \int_0^{\infty} \overline{F}(uy)\mathrm{dLN}(y; \tilde{\mu}t, \tilde{\sigma}^2 t)\mathrm{d}\lambda_t. \tag{24}$$

Moreover, it follows from (18), (21) and (14) that

$$\lim \frac{\Pr(\Delta > \varepsilon u)}{\int_0^{\infty} \Pr(X\exp\{-\tilde{\mu}t - \tilde{\sigma}W_t\} > u)\mathrm{d}\lambda_t} = 0. \tag{25}$$

By the combination of (17), (24), (25) and the arbitrariness of $\varepsilon > 0$, it follows that

$$\Psi(u) \gtrsim \int_0^\infty \int_0^\infty \overline{F}(uy)\mathrm{dLN}(y; \tilde{\mu}t, \tilde{\sigma}^2 t)\mathrm{d}\lambda_t. \tag{26}$$

Finally, the combination of (16) and (26) leads to the desired result (9).

4.3 Proof of Corollary 3.1

Rewrite the right-hand side of (9) as

$$\int_0^\infty \left[\int_1^\infty \overline{F}(uy)\mathrm{dLN}(y; \tilde{\mu}t, \tilde{\sigma}^2 t) + \int_1^\infty \overline{F}(uy)\mathrm{dLN}(y; -\tilde{\mu}t, \tilde{\sigma}^2 t)\right]\mathrm{d}\lambda_t. \tag{27}$$

By the definition of $\mathcal{R}_{-\alpha}$ in (5), for every $0 < \delta < \alpha$, there are some constants $C > 0$ and $u_0 > 0$ such that for all $uy \geqslant u \geqslant u_0$,

$$\frac{\overline{F}(uy)}{\overline{F}(u)} \leqslant Cy^{-\alpha+\delta}. \tag{28}$$

By (28), applying the dominated convergence theorem to (27) we derive the relation (10) as follows:

$$\begin{aligned}
\Psi(u) &\sim \overline{F}(u)\int_0^\infty \left[\int_1^\infty y^{-\alpha}\mathrm{dLN}(y; \tilde{\mu}t, \tilde{\sigma}^2 t) + \int_1^\infty y^{-\alpha}\mathrm{dLN}(y; -\tilde{\mu}t, \tilde{\sigma}^2 t)\right]\mathrm{d}\lambda_t \\
&= \overline{F}(u)\int_0^\infty \int_0^\infty y^{-\alpha}\mathrm{dLN}(y; \tilde{\mu}t, \tilde{\sigma}^2 t)\mathrm{d}\lambda_t \\
&= \overline{F}(u)\int_0^\infty \exp\left\{-\left(\tilde{\mu}\alpha - \frac{1}{2}\tilde{\sigma}^2\alpha^2\right)t\right\}\mathrm{d}\lambda_t = \overline{F}(u)\frac{\widehat{G}(\tilde{\mu}\alpha - \frac{1}{2}\tilde{\sigma}^2\alpha^2)}{1 - \widehat{G}(\tilde{\mu}\alpha - \frac{1}{2}\tilde{\sigma}^2\alpha^2)}.
\end{aligned}$$

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References

1 Asmussen S, Højgaard B. Approximations for finite horizon ruin probabilities in the renewal model. *Scand Actuar J*, **1999**: 106–119 (1999)

2 Avram F, Usábel M. Ruin probabilities and deficit for the renewal risk model with phase-type interarrival times. *Astin Bull*, **34**: 315–332 (2004)

3 Cossette H, Landriault D, Marceau E. Ruin probabilities in the discrete time renewal risk model. *Insurance Math Econom*, **38**: 309–323 (2006)

4 Stanford D A, Avram F, Badescu A L, et al. Phase-type approximations to finite-time ruin probabilities in the Sparre-Andersen and stationary renewal risk models. *Astin Bull*, **35**: 131–144 (2005)

5 Thorin O. On the asymptotic behavior of the ruin probability for an infinite period when the epochs of claims form a renewal process. *Scand Actuar J*, **1994**: 81–99 (1974)

6 Chen Y, Ng K W. The ruin probability of the renewal model with constant interest force and negatively dependent heavy-tailed claims. *Insurance Math Econom*, **40**: 415–423 (2007)

7 Frolova A, Kabanov Y, Pergamenshchikov S. In the insurance business risky investments are dangerous. *Finance Stoch*, **6**: 227–235 (2002)

8 Tang Q. Heavy tails of discounted aggregate claims in the continuous-time renewal model. *J Appl Probab*, **44**: 285–294 (2007)

9 Tang Q, Tsitsiashvili G. Finite- and infinite-time ruin probabilities in the presence of stochastic returns on investments. *Adv Appl Probab*, **36**: 1278–1299 (2004)

10 Tang Q, Tsitsiashvili G. Precise estimates for the ruin probability in finite horizon in a discrete-time model with heavy-tailed insurance and financial risks. *Stochastic Process Appl*, **108**: 299–325 (2003)

11 Klüppelberg C, Mikosch T. Large deviations of heavy-tailed random sums with applications in insurance and finance. *J Appl Probab*, **34**: 293–308 (1997)

12 Ng K W, Tang Q, Yan J, et al. Precise large deviations for the prospective-loss process. *J Appl Probab*, **40**: 391–400 (2003)

13 Tang Q, Su C, Jiang T, et al. Large deviations for heavy-tailed random sums in compound renewal model. *Statist Probab Lett*, **52**: 91–100 (2001)

14 Goldie C M. Subexponential distributions and dominated-variation tails. *J Appl Probab*, **15**: 440–442 (1978)

15 Bingham N H, Goldie C M, Teugels J L. Regular Variation. Cambridge: Cambridge University Press, 1987

16 Embrechts P, Klüppelberg C, Mikosch T. Modelling Extremal Events for Insurance and Finance. Berlin: Springer-Verlag, 1997

17 Dufresne D. The distribution of a perpetuity, with applications to risk theory and pension funding. *Scand Actuar J*, **1990**: 39–79 (1990)

18 Cline D B H, Samorodnitsky G. Subexponentiality of the product of independent random variables. *Stochastic Process Appl*, **49**: 75–98 (1994)

19 Gjessing H K, Paulsen J. Present value distributions with applications to ruin theory and stochastic equations. *Stochastic Process Appl*, **71**: 123–144 (1997)